RECOGNISING ACHIEVEMENT

Thursday 21 June 2012 – Afternoon

A2 GCE MATHEMATICS (MEI)

4753/01 Methods for Advanced Mathematics (C3)

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4753/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

4715660612

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

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- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Turn over

Section A (36 marks)

1 Show that $\int_1^2 \frac{1}{\sqrt{3x-2}}\,dx = \frac{2}{3}$. **[5]**

2 Solve the inequality $|2x+1| > 4$. **[3]**

3 Find the gradient at the point $(0, \ln 2)$ on the curve with equation $e^{2y} = 5 - e^{-x}$. **[4]**

4 Fig. 4 shows the curve $y = f(x)$, where $f(x) = \sqrt{1-9x^2}$, $-a \leqslant x \leqslant a$.

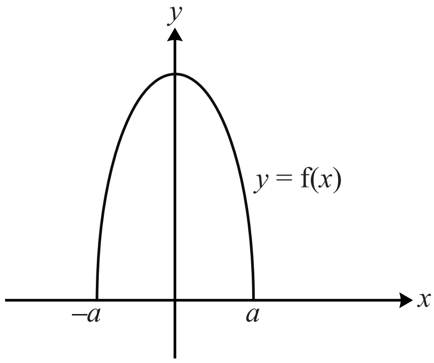

Fig. 4

(i) Find the value of a. **[2]**

(ii) Write down the range of $f(x)$. **[1]**

(iii) Sketch the curve $y = f(\frac{1}{3}x) - 1$. **[3]**

5 A termites' nest has a population of P million. P is modelled by the equation $P = 7 - 2e^{-kt}$, where t is in years, and k is a positive constant.

(i) Calculate the population when $t = 0$, and the long-term population, given by this model. **[3]**

(ii) Given that the population when $t = 1$ is estimated to be 5.5 million, calculate the value of k. **[3]**

6 Fig. 6 shows the curve $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = 2\arcsin x,\ -1 \leqslant x \leqslant 1$.

Fig. 6 also shows the curve $y = \mathrm{g}(x)$, where $\mathrm{g}(x)$ is the inverse function of $\mathrm{f}(x)$.

P is the point on the curve $y = \mathrm{f}(x)$ with x-coordinate $\frac{1}{2}$.

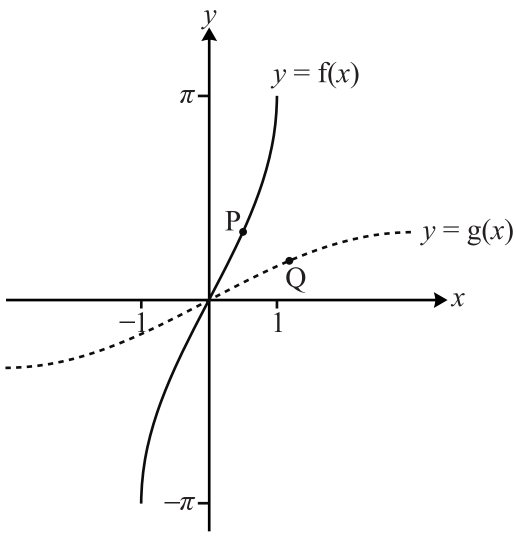

Fig. 6

(i) Find the y-coordinate of P, giving your answer in terms of π. **[2]**

The point Q is the reflection of P in $y = x$.

(ii) Find $\mathrm{g}(x)$ and its derivative $\mathrm{g}'(x)$. Hence determine the exact gradient of the curve $y = \mathrm{g}(x)$ at the point Q.

Write down the exact gradient of $y = \mathrm{f}(x)$ at the point P. **[6]**

7 You are given that $\mathrm{f}(x)$ and $\mathrm{g}(x)$ are odd functions, defined for $x \in \mathbb{R}$.

(i) Given that $\mathrm{s}(x) = \mathrm{f}(x) + \mathrm{g}(x)$, prove that $\mathrm{s}(x)$ is an odd function. **[2]**

(ii) Given that $\mathrm{p}(x) = \mathrm{f}(x)\mathrm{g}(x)$, determine whether $\mathrm{p}(x)$ is odd, even or neither. **[2]**

Section B (36 marks)

8 Fig. 8 shows a sketch of part of the curve $y = x \sin 2x$, where x is in radians.

The curve crosses the x-axis at the point P. The tangent to the curve at P crosses the y-axis at Q.

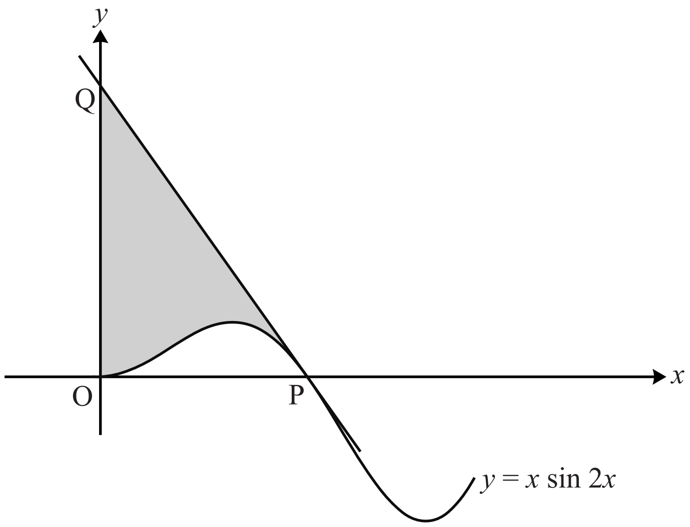

Fig. 8

(i) Find $\frac{dy}{dx}$. Hence show that the x-coordinates of the turning points of the curve satisfy the equation $\tan 2x + 2x = 0$. **[4]**

(ii) Find, in terms of π, the x-coordinate of the point P.

Show that the tangent PQ has equation $2\pi x + 2y = \pi^2$.

Find the exact coordinates of Q. **[7]**

(iii) Show that the exact value of the area shaded in Fig. 8 is $\frac{1}{8}\pi(\pi^2 - 2)$. **[7]**

9 Fig. 9 shows the curve $y = \mathrm{f}(x)$, which has a y-intercept at P(0, 3), a minimum point at Q(1, 2), and an asymptote $x = -1$.

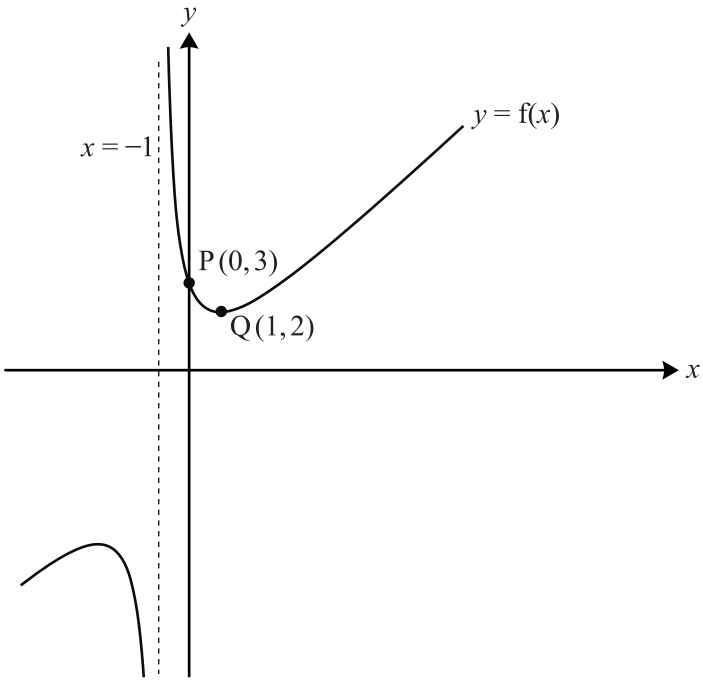

Fig. 9

(i) Find the coordinates of the images of the points P and Q when the curve $y = \mathrm{f}(x)$ is transformed to

(A) $y = 2\mathrm{f}(x)$,

(B) $y = \mathrm{f}(x + 1) + 2$. **[4]**

You are now given that $\mathrm{f}(x) = \dfrac{x^2 + 3}{x + 1}$, $x \neq -1$.

(ii) Find $\mathrm{f}'(x)$, and hence find the coordinates of the other turning point on the curve $y = \mathrm{f}(x)$. **[6]**

(iii) Show that $\mathrm{f}(x - 1) = x - 2 + \dfrac{4}{x}$. **[3]**

(iv) Find $\displaystyle\int_a^b \left(x - 2 + \frac{4}{x}\right) \mathrm{d}x$ in terms of a and b.

Hence, by choosing suitable values for a and b, find the exact area enclosed by the curve $y = \mathrm{f}(x)$, the x-axis, the y-axis and the line $x = 1$. **[5]**

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OCR
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4753/01 Methods for Advanced Mathematics (C3)

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Candidates answer on this Printed Answer Book.

OCR supplied materials:
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Candidate forename		Candidate surname	

Centre number						Candidate number				

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Section A (36 marks)

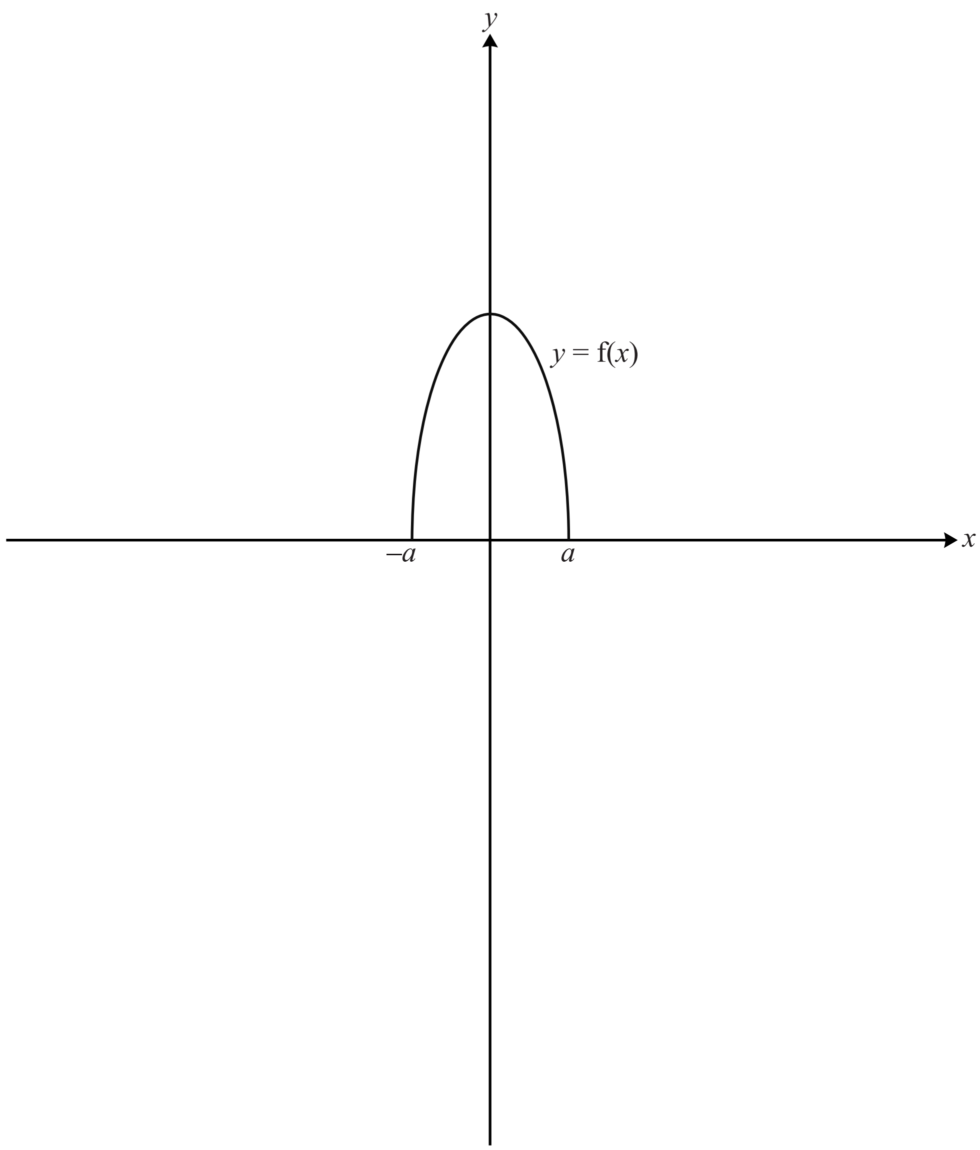
1	

2

3

(answer space continued overleaf)

3	(continued)
4 (i)	

4 (ii)	
4 (iii)	

5 (i)	
5 (ii)	

6 (i)	
6 (ii)	
	(answer space continued overleaf)

6 (ii)	(continued)

7 (i)	
7 (ii)	

Section B (36 marks)

8 (i)	
8 (ii)	
	(answer space continued on next page)

8 (ii)	**(continued)**

8 (iii)	
	(answer space continued on next page)

(answer space continued on next page)

8 (iii)	**(continued)**
9 (i)(*A*)	
9 (i)(*B*)	

9 (ii)	

9 (iii)	
9 (iv)	
	(answer space continued overleaf)

9 (iv)	(continued)

RECOGNISING ACHIEVEMENT

GCE

Mathematics (MEI)

Advanced GCE

Unit **4753:** Methods for Advanced Mathematics

Mark Scheme for June 2012

Oxford Cambridge and RSA Examinations

OCR (Oxford Cambridge and RSA) is a leading UK awarding body, providing a wide range of qualifications to meet the needs of candidates of all ages and abilities. OCR qualifications include AS/A Levels, Diplomas, GCSEs, OCR Nationals, Functional Skills, Key Skills, Entry Level qualifications, NVQs and vocational qualifications in areas such as IT, business, languages, teaching/training, administration and secretarial skills.

It is also responsible for developing new specifications to meet national requirements and the needs of students and teachers. OCR is a not-for-profit organisation; any surplus made is invested back into the establishment to help towards the development of qualifications and support, which keep pace with the changing needs of today's society.

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

Any enquiries about publications should be addressed to:

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Annotations and abbreviations

Annotation in scoris	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	

Other abbreviations in mark scheme	Meaning
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions for GCE Mathematics (MEI) Pure strand

a. Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b. An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c. The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d. When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e. The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f. Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g. Rules for replaced work

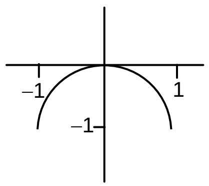
If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.
If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h. For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question			Answer	Marks	Guidance	
1			$\int_1^2 \frac{1}{\sqrt{3x-2}}\,\mathrm{d}x = \left[\frac{2}{3}(3x-2)^{1/2}\right]_1^2$	M1 A2	$[k\,(3x-2)^{1/2}]$ $k = 2/3$	
			$= \frac{2}{3}.2 - \frac{2}{3}.1$ $= 2/3$*	M1dep A1	substituting limits dep 1st M1 NB **AG**	
			OR $u = 3x - 2 \Rightarrow \mathrm{d}u/\mathrm{d}x = 3$ $\Rightarrow \int_1^2 \frac{1}{\sqrt{3x-2}}\,\mathrm{d}x = \int_1^4 \frac{1}{\sqrt{u}}.\frac{1}{3}\,\mathrm{d}u$	M1 A1	$\int \frac{1}{\sqrt{u}}$ $\times\ 1/3\ (\mathrm{d}u)$	or $w^2 = 3x - 2 \Rightarrow \int \frac{1}{w}$ $\times\ 2/3\ w\ (\mathrm{d}w)$
			$= \left[\frac{2}{3}u^{1/2}\right]_1^4 = \frac{2}{3}.2 - \frac{2}{3}.1$	A1	$\left[\frac{2}{3}u^{1/2}\right]$ o.e.	$\left[\frac{2}{3}w\right]$
			$= 2/3$*	M1dep A1 **[5]**	substituting correct limits dep 1st M1 NB **AG**	upper – lower, 1 to 4 for u or 1 to 2 for w or substituting back (correctly) for x and using 1 to 2
2			$\lvert 2x+1 \rvert > 4$ $\Rightarrow\ 2x + 1 > 4,\ x > 3/2$ or $2x + 1 < -4$, $x < -2\frac{1}{2}$	B1 M1 A1 **[3]**	$x > 3/2$ mark final ans; if from $\lvert 2x \rvert > 3$ B0 o.e., e.g. $-(2x + 1) > 4$ (or $2x + 1 = -4$) if $\lvert 2x+1 \rvert < -4$, M0 $x < -2\frac{1}{2}$ mark final ans allow 3 for correct unsupported answers	by squaring: $4x^2 + 4x - 15 >$ (or $=$) 0 M1 $x > 3/2$ A1 $x < -2\frac{1}{2}$ A1 Penalise $\geq$ and $\leq$ once only $3/2 < x < -2\frac{1}{2}$ SC2 (mark final ans)
3			$e^{2y} = 5 - e^{-x}$ $\Rightarrow\ 2e^{2y}\frac{\mathrm{d}y}{\mathrm{d}x} = e^{-x}$ $\Rightarrow\ \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{e^{-x}}{2e^{2y}}$	B1 B1	$2e^{2y}\frac{\mathrm{d}y}{\mathrm{d}x} = \ldots$ $= e^{-x}$	or $y = \ln\sqrt{(5 - e^{-x})}$ o.e (e.g. $\frac{1}{2}\ln(5 - e^{-x})$)B1 $\Rightarrow \mathrm{d}y/\mathrm{d}x = e^{-x}/[2(5 - e^{-x})]$ o.e. B1 (but must be correct)
			At (0, ln2) $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{e^0}{2e^{2\ln 2}}$ $= \frac{1}{8}$	M1dep A1cao **[4]**	substituting $x = 0$, $y = \ln 2$ into their $\mathrm{d}y/\mathrm{d}x$ dep 1st B1 – allow one slip	or substituting $x = 0$ into their correct $\mathrm{d}y/\mathrm{d}x$

Question			Answer	Marks	Guidance	
4	(i)		$1 - 9a^2 = 0$ $\Rightarrow a^2 = 1/9 \Rightarrow a = 1/3$	M1 A1 **[2]**	or $1 - 9x^2 = 0$ or 0.33 or better $\sqrt{(1/9)}$ is A0	$\sqrt{(1 - 9a^2)} = 1 - 3a$ is M0 not $a = \pm\ 1/3$ nor $x = 1/3$
4	(ii)		Range $0 \le y \le 1$	B1 **[1]**	or $0 \le \mathrm{f}(x) \le 1$ or $0 \le \mathrm{f} \le 1$, not $0 \le x \le 1$ $0 \le y \le \sqrt{1}$ is B0	allow also [0,1], or 0 to 1 inclusive, but not 0 to 1 or (0,1)
4	(iii)			M1 M1 A1 **[3]**	curve goes from $x = -3a$ to $x = 3a$ (or -1 to 1) vertex at origin curve, 'centre' $(0,-1)$, from $(-1, -1)$ to $(1, -1)$ (y-coords of -1 can be inferred from vertex at O and correct scaling)	must have evidence of using s.f. 3 allow also if s.f.3 is stated and stretch is reasonably to scale allow from $(-3a, -1)$ to $(3a, -1)$ provided $a = 1/3$ or $x = [\pm]\ 1/3$ in (i) A0 for badly inconsistent scale(s)
5	(i)		When $t = 0$, $P = 7 - 2 = 5$, so 5 (million) In the long term $\mathrm{e}^{-kt} \to 0$ So long-term population is 7 (million)	B1 M1 A1 **[3]**	 allow substituting a large number for t (for both marks)	 allow 7 unsupported
5	(ii)		$P = 7 - 2\mathrm{e}^{-kt}$ When $t = 1$, $P = 5.5$ $\Rightarrow \quad 5.5 = 7 - 2\mathrm{e}^{-k}$ $\Rightarrow \quad \mathrm{e}^{-k} = (7 - 5.5)/2 = 0.75$ $\Rightarrow \quad -k = \ln((7 - 5.5)/2)$ $\Rightarrow \quad k = 0.288$ (3 s.f.)	 M1 M1 A1 **[3]**	 re-arranging and anti-logging – allow 1 slip (e.g. arith of $7 - 5.5$, or k for $-k$) or $\ln 2 - k = \ln 1.5$ o.e. 0.3 or better allow $\ln(4/3)$ or $-\ln(3/4)$ if final ans	 but penalise negative lns, e.g. $\ln(-1.5) = \ln(-2) - k$ rounding from a correct value of $k = 0.2876820725\ldots$, penalise truncation, and incorrect work with negatives

Question			Answer	Marks	Guidance	
6	(i)		$y = 2 \arcsin \frac{1}{2} = 2 \times \pi/6$ $= \pi/3$	M1 A1 **[2]**	$y = 2 \arcsin \frac{1}{2}$ must be in terms of π − can isw approximate answers	1.047… implies M1
6	(ii)		$y = 2 \arcsin x$ $\quad x \leftrightarrow y$ $\Rightarrow$ $x = 2 \arcsin y$ $\Rightarrow$ $x/2 = \arcsin y$ $\Rightarrow$ $y = \sin(x/2)$ [so $g(x) = \sin(x/2)$] $\Rightarrow$ $dy/dx = \frac{1}{2}\cos(\frac{1}{2}x)$ At Q, $x = \pi/3$ $\Rightarrow$ $dy/dx = \frac{1}{2}\cos \pi/6 = \frac{1}{2}\sqrt{3}/2 = \sqrt{3}/4$ $\Rightarrow$ gradient at P = $4/\sqrt{3}$	M1 A1 A1cao M1 A1 B1 ft **[6]**	or $y/2 = \arcsin x$ but must interchange x and y at some stage substituting their $\pi/3$ into their derivative must be exact, with their $\cos(\pi/6)$ evaluated o.e. e.g. $4\sqrt{3}/3$ but must be exact ft their $\sqrt{3}/4$ unless 1	or $f'(x) = 2/\sqrt{(1-x^2)}$ $f'(\frac{1}{2}) = 2/\sqrt{\frac{3}{4}} = 4/\sqrt{3}$ cao
7	(i)		$s(-x) = f(-x) + g(-x)$ $= -f(x) + -g(x)$ $= -(f(x) + g(x))$ $= -s(x)$ (so s is odd)	M1 A1 **[2]**	must have $s(-x) = \ldots$	
7	(ii)		$p(-x) = f(-x)g(-x)$ $=(-f(x)) \times (-g(x))$ $= f(x)g(x) = p(x)$ so p is even	M1 A1 **[2]**	must have $p(-x) = \ldots$ Allow SC1 for showing that $p(-x) = p(x)$ using two specific odd functions, but in this case they must still show that p is even	e.g. $f(x) = x$, $g(x) = x^3$, $p(x) = x^4$ $p(-x) = (-x)^4 = x^4 = p(x)$, so p even condone f and g being the same function

Question			Answer	Marks	Guidance	
8	(i)		$\frac{dy}{dx} = \sin 2x + 2x\cos 2x$	M1	d/dx(sin 2x) = 2cos 2x soi	can be inferred from dy/dx = 2x cos 2x
				A1	cao, mark final answer	e.g. dy/dx = tan 2x + 2x is A0
			dy/dx = 0 when sin 2x + 2x cos 2x = 0	M1	equating their derivative to zero, **provided it has two terms**	
			$\Rightarrow \frac{\sin 2x + 2x\cos 2x}{\cos 2x} = 0$			
			$\Rightarrow$ tan 2x + 2x = 0 *	A1	must show evidence of division by cos 2x	
				[4]		
8	(ii)		At P, x sin 2x = 0	M1		Finding $x = \pi/2$ using the given line equation is M0
			$\Rightarrow$ sin 2x = 0, 2x = (0), $\pi \Rightarrow x = \pi/2$	A1	$x = \pi/2$	
			At P, dy/dx = sin π + 2(π/2) cos π = $-\pi$	B1 ft	ft their $\pi/2$ and their derivative	
			Eqn of tangent: $y - 0 = -\pi(x - \pi/2)$	M1	substituting 0, their $\pi/2$ and their $-\pi$ into $y - y_1 = m(x - x_1)$	or their $-\pi$ into $y = mx+c$, and then evaluating c: $y = (-\pi)x + c$,
			$\Rightarrow y = -\pi x + \pi^2/2$			0 = $(-\pi)(\pi/2) + c$ M1
			$\Rightarrow 2\pi x + 2y = \pi^2$ *	A1	**NB AG**	$\Rightarrow c = \pi^2/2$
						$\Rightarrow y = -\pi x + \pi^2/2 \Rightarrow 2\pi x + 2y = \pi^2$ *A1
			When $x = 0$, $y = \pi^2/2$, so Q is (0, $\pi^2/2$)	M1A1	can isw inexact answers from $\pi^2/2$	
				[7]		
8	(iii)		Area = triangle OPQ – area under curve	M1	soi (or area under PQ – area under curve	area under line may be expressed in integral form
			Triangle OPQ = ½ × π/2 × π^2/2 [= π^3/8]	B1cao	allow art 3.9	or using integral:
						$\int_0^{\pi/2}(\frac{1}{2}\pi^2 - \pi x)\,dx = \left[\frac{1}{2}\pi^2 x - \frac{1}{2}\pi x^2\right]_0^{\pi/2} = \frac{\pi^3}{4} - \frac{\pi^3}{8} [= \frac{\pi^3}{8}]$
			Parts: $u = x$, dv/dx = sin 2x	M1	condone $v = k$ cos2x soi	v can be inferred from their 'uv'
			du/dx = 1, v = –½ cos2x			
			$\int_0^{\pi/2} x\sin 2x\,dx = \left[-\frac{1}{2}x\cos 2x\right]_0^{\pi/2} - \int_0^{\pi/2} -\frac{1}{2}\cos 2x\,dx$	A1ft	ft their v = –½ cos2x, ignore limits	
			$= \left[-\frac{1}{2}x\cos 2x + \frac{1}{4}\sin 2x\right]_0^{\pi/2}$	A1	[– ½ x cos 2x + ¼ sin 2x] o.e., must be correct at this stage, ignore limits	
			$= -\frac{1}{4}\pi\cos\pi + \frac{1}{4}\sin\pi - (-0\cos 0 + \frac{1}{4}\sin 0) = \frac{1}{4}\pi[-0]$	A1cao	(so dep previous A1)	
			So shaded area = $\pi^3/8 - \pi/4 = \pi(\pi^2 - 2)/8$*	A1	**NB AG** must be from fully correct work	
				[7]		

Question			Answer	Marks	Guidance	
9	(i)		(A) (0, 6) and (1, 4) (B) (−1, 5) and (0, 4)	B1B1 B1B1 **[4]**	Condone P and Q incorrectly labelled (or unlabelled)	
9	(ii)		$f'(x)=\dfrac{(x+1).2x-(x^2+3).1}{(x+1)^2}$ $f'(x) = 0 \Rightarrow 2x\,(x + 1) - (x^2 + 3) = 0$ $\Rightarrow x^2 + 2x - 3 = 0$ $\Rightarrow (x - 1)(x + 3) = 0$ $\Rightarrow x = 1$ or $x = -3$ When $x = -3$, $y = 12/(-2) = -6$ so other TP is (−3, −6)	M1 A1 M1 A1dep B1B1cao **[6]**	Quotient or product rule consistent with their derivatives, condone missing brackets correct expression their derivative = 0 obtaining correct quadratic equation (soi) dep 1st M1 but withhold if denominator also set to zero must be from correct work (but see note re quadratic)	PR: $(x^2+3)(-1)(x+1)^{-2} + 2x(x+1)^{-1}$ If formula stated correctly, allow one substitution error. condone missing brackets if subsequent working implies they are intended Some candidates get $x^2 + 2x + 3$, then realise this should be $x^2 + 2x - 3$, and correct back, but not for every occurrence. Treat this sympathetically. Must be supported, but −3 could be verified by substitution into correct derivative
9	(iii)		$f(x-1)=\dfrac{(x-1)^2+3}{x-1+1}$ $=\dfrac{x^2-2x+1+3}{x-1+1}$ $=\dfrac{x^2-2x+4}{x}=x-2+\dfrac{4}{x}$ *	M1 A1 A1 **[3]**	substituting $x - 1$ for both x's in f NB **AG**	allow 1 slip for M1
9	(iv)		$\int_a^b (x-2+\frac{4}{x})\,dx=\left[\frac{1}{2}x^2-2x+4\ln x\right]_a^b$ $=(\frac{1}{2}b^2-2b+4\ln b)-(\frac{1}{2}a^2-2a+4\ln a)$ Area is $\int_0^1 f(x)\,dx$ So taking a = 1 and b = 2 area = (2 − 4 + 4ln 2) − (½ − 2 + 4ln 1) = 4 ln 2 − ½	B1 M1 A1 M1 A1 cao **[5]**	$\left[\frac{1}{2}x^2-2x+4\ln x\right]$ F(b) − F(a) condone missing brackets oe (mark final answer) must be simplified with ln 1 = 0	F must show evidence of integration of at least one term or f(x) = x + 1 − 2 + 4/(x+1) $A=\int_0^1 f(x)\,dx=\left[\frac{1}{2}x^2-x+4\ln(1+x)\right]_0^1$ M1 = ½ − 1 + 4ln 2 = 4ln2 − ½ A1

4753 Methods for Advanced Mathematics (C3 Written Examination)

General Comments

This proved to be an accessible paper, and raw marks were consequently higher than in recent examinations, with quite a few candidates managing to score full marks. Few candidates had difficulty completing the paper in the allotted time. Many scripts were well presented, though the longer part questions, such as 8(iii), sometimes suffered from a lack of systematic presentation. There was evidence in some scripts of attempts to 'fiddle' expressions to achieve given results – the resulting inconsistency often lost marks.

Comments on Individual Questions

1) This was a straightforward starter question, for which many candidates scored full marks. The most popular strategy was to use the substitution $u = 3x – 2$, and candidates were generally adept at replacing dx with 1/3 du, integrating correctly and substituting correct limits. In a few cases ln $u^{½}$ was obtained after integration. The substitution $u = (3x – 2)^{½}$ was less common and caused greater difficulty. Relatively few students attempted to integrate directly without substitution, but those that did often succeeded, and gained the 5 marks with ease.

2) Most candidates scored at least two of the marks. A very common error was failing to change the inequality sign when proceeding from $−2x > 5$ to $x < -5/2$. A few sketched the graph of $y = |2x + 1|$ to solve the problem. Other errors seen occasionally were $|2x + 1| > −4$ and $|2x| > 3$. Even some good candidates do not seem to appreciate that nesting the two inequalities as $-5/2 > x > 3/2$ is incorrect.

3) This implicit differentiation was generally well done. The most common error was $d/dx(e^{-x}) = e^{x}$ instead of e^{-x}. Some candidates re-arranged the original equation correctly to give $y = ½ \ln(5 – e^{−x})$, though log errors were quite common here; however, many went on from here by differentiating this incorrectly.

4) (i) This was usually correct, though leaving the final answer as $x = 1/3$ was quite common, and some candidates left their answer as $\sqrt{(1/9)}$, or gave $a = \pm 1/3$.

(ii) The domain was frequently given instead of the range. Other answers scoring zero included $0 \le x \le 1$, $y \le 1$, 0 to 1 (which does not settle the inclusion of the endpoints), and 1.

(iii) Many gained only the method marks because they omitted to indicate the domain or range on their sketch. Others did not indicate the x-coordinates of the endpoints. Some stretches looked more like enlargements. To get the final 'A' mark, we required the axes to have approximately the same scale.

5) (i) The initial value of $P = 5$ was answered correctly by nearly all candidates, but the long-term value defeated some.

(ii) This part was equally well answered. The most common errors were to re-arrange the initial equation incorrectly or to take logarithms of each side incorrectly, e.g. $\ln 5.5 = \ln 7 – \ln 2 \times k$.

6) (i) This was generally answered successfully, with only a few failing to give the exact value $\pi/3$.

(ii) (ii) Most candidates successfully found the inverse function, but ½ sin x was occasionally seen. Once that hurdle was crossed, most differentiated sin ½ x correctly, though cos(½x) and 2 cos(½x) were seen. The substitution of $x = \pi/3$ was usually correct, though a small number used $x = 1$. The gradient at P was usually the reciprocal of that at Q, with $-1/m$ (instead of $1/m$) being the most common error. A few candidates differentiated f(x) directly, often with success.

7) As often happens when candidates are attempting to prove this type of result, many work with both sides at once, producing repetitive and confused solutions which did not always receive the benefit of the examiner's doubt. Most seemed to know about odd and even functions, but often failed to express this correctly, for example writing statements such as f(x) = −f(x).

(i) (i) This two-line proof defeated most candidates, mainly because they failed to start off with s(-x) = f(-x) + g(-x) = … Starting with f(x) = −f(−x) also made life harder than necessary. A few candidates used the functions f and g from the previous question rather than treating these as general functions.

(ii) (ii) Quite a few candidates recognised that p(x) was even, but we wanted to see a proper argument to justify this, and, as in part (i), this really required them to start p(-x) = f(-x)g(-x).

8) (i) The vast majority differentiated correctly - though 2xcos2x was seen occasionally - and equated their derivative to zero. Most then succeeded in dividing by cos 2x to arrive at the required result. Some candidates, however, divided before equating the derivative to zero, and gave the derivative as 2x + tan 2x.

(ii) Most candidates solved x sin 2x = 0 to obtain x = π/2 at P. The derivative was then required to obtain the gradient of the tangent and hence its equation, but some used the given tangent equation itself to find the gradient. The last part was successfully completed by nearly all candidates, with the given tangent equation being used to obtain the correct y-coordinate at Q of $\pi^2/2$.

(iii) Most candidates attempted find the area of the triangle and the area under the curve, though a clear statement of method was not always given. Quite a few candidates attempted to find the triangle area by integration, and came unstuck in the process. The area under the curve was generally recognised as integration by parts, but marks were lost through incorrect v', or mistakes with signs. Some tried to combine both integrals (for line and curve), and got into a muddle by stock-piling negative signs, rather than simplifying these on a step-by-step basis. Nevertheless, good candidates had little trouble in supplying a fluent solution.

9) (i) The majority of candidates obtained full marks; part (A) was not as well answered as (B): sometimes marks were lost through stretching horizontally rather than vertically.

(ii) This was all relatively routine work which good candidates had little trouble with; however, several candidates made slips in expanding the bracket in the numerator of the quotient rule. This was a costly error, as were errors in the quotient rule such as uv'−vu' in the numerator.

(iii) Most candidates were successful, guided by the given answer, though a few found f(x) + 1.

(iv) Most integrated the function correctly, though ¼ ln(x) was seen occasionally. The question asked candidates to give the answer in terms of a and b, but some omitted this. The final answer required candidates to realise that a = 1 and b = 2, rather than 0 and 1 (notwithstanding the appearance of ln 0 in the lower limit), but this was spotted by only the better candidates.

OCR (Oxford Cambridge and RSA Examinations)
1 Hills Road
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CB1 2EU

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GCE Mathematics (MEI)

			Max Mark	90% cp	a	b	c	d	e	u
4753/01	(C3) MEI Methods for Advanced Mathematics with Coursework: Written Paper	Raw	72	66	60	53	47	41	34	0
4753/02	(C3) MEI Methods for Advanced Mathematics with Coursework: Coursework	Raw	18	16	15	13	11	9	8	0
4753/82	(C3) MEI Methods for Advanced Mathematics with Coursework: Carried Forward Coursework Mark	Raw	18	16	15	13	11	9	8	0
4753	(C3) MEI Methods for Advanced Mathematics with Coursework	UMS	100	90	80	70	60	50	40	0
4754/01	(C4) MEI Applications of Advanced Mathematics	Raw	90	73	65	57	50	43	36	0
		UMS	100	90	80	70	60	50	40	0
4756/01	(FP2) MEI Further Methods for Advanced Mathematics	Raw	72	66	61	53	46	39	32	0
		UMS	100	90	80	70	60	50	40	0
4757/01	(FP3) MEI Further Applications of Advanced Mathematics	Raw	72	61	54	47	40	34	28	0
		UMS	100	90	80	70	60	50	40	0
4758/01	(DE) MEI Differential Equations with Coursework: Written Paper	Raw	72	68	63	57	51	45	39	0
4758/02	(DE) MEI Differential Equations with Coursework: Coursework	Raw	18	16	15	13	11	9	8	0
4758/82	(DE) MEI Differential Equations with Coursework: Carried Forward Coursework Mark	Raw	18	16	15	13	11	9	8	0
4758	(DE) MEI Differential Equations with Coursework	UMS	100	90	80	70	60	50	40	0
4762/01	(M2) MEI Mechanics 2	Raw	72	65	58	51	44	38	32	0
		UMS	100	90	80	70	60	50	40	0
4763/01	(M3) MEI Mechanics 3	Raw	72	67	63	56	50	44	38	0
		UMS	100	90	80	70	60	50	40	0
4764/01	(M4) MEI Mechanics 4	Raw	72	63	56	49	42	35	29	0
		UMS	100	90	80	70	60	50	40	0
4767/01	(S2) MEI Statistics 2	Raw	72	66	61	55	49	43	38	0
		UMS	100	90	80	70	60	50	40	0
4768/01	(S3) MEI Statistics 3	Raw	72	65	58	51	44	38	32	0
		UMS	100	90	80	70	60	50	40	0
4769/01	(S4) MEI Statistics 4	Raw	72	63	56	49	42	35	28	0
		UMS	100	90	80	70	60	50	40	0
4772/01	(D2) MEI Decision Mathematics 2	Raw	72	62	56	50	44	39	34	0
		UMS	100	90	80	70	60	50	40	0
4773/01	(DC) MEI Decision Mathematics Computation	Raw	72	52	46	40	34	29	24	0
		UMS	100	90	80	70	60	50	40	0
4777/01	(NC) MEI Numerical Computation	Raw	72	63	55	47	39	32	25	0
		UMS	100	90	80	70	60	50	40	0